

# Temperature and Radiation

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The three main methods of heat transfer resulting in change of temperature are conduction, convection and radiation. In conduction, energy is transferred by physical contact, like when one burns a finger while touching a hot pot. In convection, energy transfer occurs by fluid motion like water boiling in a kettle. In radiation, energy is transferred by the absorption or emission of electromagnetic radiation like the warmth from the Sun. This paper will illustrate some examples of how radiation determines temperature of objects in space and on Earth.

## Electromagnetic Radiation

All bodies with a temperature greater than absolute zero radiate energy. Absolute zero is the temperature at which there is no molecular or atomic random motion. It's denoted by 0 Kelvin degrees, which is equivalent to -273.15° C or -459.67° F. Late in the nineteenth century, Stefan experimentally and Boltzmann theoretically developed a relationship between the temperature of a body and the amount of power it radiates.

To determine outgoing radiation power, we utilize the Stefan-Boltzmann Law:

$$P = A\epsilon\sigma T^4 \quad (1)$$

Where P (watts) is the radiated power from a body of area A (m<sup>2</sup>) at temperature T (K).  
ε is emissivity, a dimensionless number between 0 and 1 that determines the efficiency of a body to radiate and absorb energy. A perfect radiator and absorber has an emissivity of 1. Soil, ice, rock, asphalt and human skin have emissivities slightly less than 1.  
σ is the Stefan-Boltzmann constant,  $5.67 \times 10^{-8} \text{ Wm}^{-2}\text{T}^{-4}$   
T is the body temperature in Kelvin.

So if absolute temperature (in Kelvin degrees) doubles, radiated power increases by a factor of sixteen. Also, changes in temperature alter radiation peak wavelengths. Temperature increases move peak radiation to smaller wavelengths and vice-versa.

## Equilibrium and Solid Angles

Assume we have several bodies of different temperatures and we want to determine temperature of a specific body from their radiation. At steady state or equilibrium, radiation received must equal radiation emitted. The amount of radiation received depends on the emitting body's temperature, its size and its distance to the receiving body. Size and distance are quantified by calculating the solid angle. Solid angles are defined as the area of the emitting body divided by distance squared from the receiving body, with units called steradians:

$$\text{Solid angle in steradians, } \Omega = \frac{\text{area}}{\text{distance}^2} \quad (2)$$

For example, the solid angle of a body in space absorbing cosmic microwave background (CMB) radiation which comes from all directions is: (area of a sphere perceived from its

center)/(radius<sup>2</sup>) =  $4\pi r^2 / r^2 = 4\pi$  steradians. The solid angle of a body receiving radiation from the Sun at 1 astronomical unit (AU) distance is: (solar disk area)/(distance<sup>2</sup>) or

$$\Omega = \frac{area_{sun}}{distance_{sun}^2} = \frac{\pi(7 \times 10^5)^2}{(1.5 \times 10^8)^2} = 6.84 \times 10^{-5} \text{ steradians, where the Sun's radius is } \sim 700,000 \text{ km and}$$

an astronomical unit is  $\sim 150$  million kilometers.

Let's assume we have a body in space receiving radiation from other sources. It will in turn radiate power based on its temperature. At equilibrium, radiation absorbed will equal radiation emitted:

$$\Omega_{body} T_{body}^4 = \Omega_{cmb} T_{cmb}^4 + \Omega_{sun} T_{sun}^4 + others \quad (3)$$

We can solve for  $T_{body}$  by inserting the  $\Omega$  and T factors in the equation. For example,  $T_{cmb} = \sim 2.7$  K,  $T_{sun} = \sim 5800$  K,  $\Omega_{body} = 4\pi$  for effective temperature,  $\Omega_{cmb} = 4\pi$ , and  $\Omega_{sun} = 6.84 \times 10^{-5}$  steradians at one astronomical unit (AU).

### Temperatures Due to the Sun

In space, the major factor for temperatures of solar system planets and asteroids is the contribution from the Sun. However, other sources might include radiation from the cosmic microwave background, other nearby bodies, tidal effects, or internal sources of heat from elements with long term radioactivity. We'll assume ideal conditions: emissivity = 1 for all bodies and no radiation attenuation through space.

For example, assume a body orbits the Sun with a semi-major axis of 1 AU (150 million km). The Sun's "surface" temperature is about 5800 K. We'll assume the body's albedo is zero and it has no greenhouse atmosphere. Recall that albedo is the fraction of radiation reflected back to space. CMB radiation will also be ignored. We can calculate the body's effective temperature as follows:

$$4\pi T_{eff}^4 = 6.84 \times 10^{-5} (5800)^4 \quad (4)$$

$$T_{eff} = \left( \frac{(6.84 \times 10^{-5})(5800)^4}{4\pi} \right)^{1/4} = 280K \quad (5)$$

If the body has an albedo A, and has a greenhouse factor G, we can calculate its effective temperature as follows:

$$4\pi(1 - G)T_{avg}^4 = 6.84 \times 10^{-5} (1 - A)(5800)^4 \quad (6)$$

For example, Earth's albedo, has a value of about 0.3 due to clouds. Its greenhouse factor, which reduces outgoing radiation is about 0.4, mainly due to water vapor in its atmosphere. Earth's predicted average temperature is:

$$T_{avg} = \left( \frac{1 - A}{1 - G} \right)^{1/4} \left( \frac{(6.84 \times 10^{-5})(5800)^4}{4\pi} \right)^{1/4} = \left( \frac{1 - A}{1 - G} \right)^{1/4} T_{eff} = \left( \frac{0.7}{0.6} \right)^{1/4} (280K) = 291K \quad (7)$$

Earth's actual average temperature is 288 K.

A simplified formula to determine the effective temperature of a body orbiting the Sun at a semi-major axis in AU of  $D_{AU}$  is:

$$T = \left( \frac{1-A}{1-G} \right)^{1/4} \frac{280K}{\sqrt{D_{AU}}} \quad (8)$$

For example, the dwarf planet Eris has a semi-major axis of 68 AU. Assuming no albedo or greenhouse effects, its effective temperature would be  $280K/\sqrt{68} = 34K$ .

There have been many exoplanets discovered in the past decade. Ignoring albedo and greenhouse effects, a formula for the effective temperature of an exoplanet is:

$$T_{exo} = \frac{1}{2} \sqrt{\frac{diameter_{star}}{distance_{star}}} x T_{star} \quad (9)$$

Instead of effective temperatures, we might want to determine the maximum or sub-solar temperature of a body orbiting the Sun. In equilibrium temperature calculations, a body radiates in all directions, the area of a sphere. However, for maximum temperature calculations, radiation is only important facing the Sun, the area of a circle. This would require making the calculation by substituting an area solid angle of  $\pi$  steradians for the spherical  $4\pi$  steradians solid angle used in equilibrium temperatures. The maximum temperature on the Moon would be:

$$\pi T_{max}^4 = (6.84 \times 10^{-5})(5800)^4 \quad (10)$$

$$T_{max} = \left( \frac{(6.84 \times 10^{-5})(5800)^4}{\pi} \right)^{1/4} = 2^{1/2} T_{eff} = 1.41 \times 280K = 396K \quad (11)$$

This is equivalent to 253° F. Sub-solar or maximum temperatures are ~41% greater than effective temperatures.

## Temperature from Radiation on Earth

In this section, we'll explore maximum temperature changes when hot or cold objects are encountered when surrounded by ambient temperature conditions. Accuracy is compromised by ignoring radiation losses that could be caused by the atmosphere.

### Nuclear Fireball

Assume an observer is 100 km from a nuclear explosion, a one megaton device whose fireball's temperature is 10,000 K and diameter is one kilometer. Ambient temperature is 300 K (80° F). What is the temperature facing the device at the observer's location?

The solid angle from the observer's perspective to the nuclear device is  $\pi$  steradians, a situation analogous to subsolar temperature planetary calculations above. The device's solid angle is  $\frac{\pi(0.5)^2}{100^2} = 7.85 \times 10^{-5}$  steradians. The observer's temperature is affected by ambient and device temperatures:

$$\pi T_{observer}^4 = \pi(300)^4 + (7.85 \times 10^{-5})(10000)^4 \quad (12)$$

This results in a temperature of 713 K, or 824° F, a change of 413° C or 744° F, which would cause fires and severe burns for anyone facing the blast. However, anything in the shade of the nuclear device would only be exposed to the ambient 80° F temperature.

### Encountering an Iceberg

A ship passes 0.5 km from a 1 km long, 100m high iceberg. The iceberg's temperature is 273 K (32° F) and the ambient temperature is 300 K. What is the temperature drop on the ship's side facing the iceberg as it passes by?

The iceberg's perceived solid angle is  $\frac{(1 \times 0.1)}{0.5^2} = 0.4$  steradians.

This is a significant percentage of the ambient  $\pi$  solid angle. Therefore, a reduction of 0.4 steradians will be required.

$$\pi T_{ship}^4 = (\pi - 0.4)(300)^4 + 0.4(273)^4 \quad (13)$$

As it moves by the iceberg, the ship's temperature would gradually drop to 297 K (75° F). The drop of 5° F would likely be noticed by passengers on the ship's side facing the iceberg. The other side of the ship would remain at 80° F.

## References

[http://en.wikipedia.org/wiki/Electromagnetic\\_spectrum](http://en.wikipedia.org/wiki/Electromagnetic_spectrum)

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*See related tutorial on this website titled, ["The Dewing or Frosting of Telescope Optics"](#).*